

Information gain versus interference in Bohr's principle of complementarity

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Based on modern quantum measurement theory, we use Zurek's "triple model" to study, from the viewpoint of quantum information theory, the wave and particle nature of a photon in a symmetric Mach-Zehnder interferometer. In the process of quantum measurement, the state of both the system and the detector is not an entangled state but a correlated state. We find that the information gain about the photon is related to the correlations (including classical and quantum correlations) between the photon and the detector. We also derive the relationship between the information gain and the fringe visibility. We find that the classical correlations remain consistent with the path distinguishability and can be used to describe the particle-like property of the photon. Quantum correlations are not exactly the same as fringe visibility, but both can represent the quantum coherence of the photon. Finally, we provide an analytical expression for quantum correlations of one type of two-qubit separable states.

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I. INTRODUCTION

Unlike classical particles, a quantum system behaves either as a particle or as a wave. This property is called wave-particle duality, which is one of the famous intriguing feature of quantum mechanics. Quantum properties which are equally real but mutually exclusive are called complementary [1, 2]. Wave-particle duality is well described in Bohr's complementarity principle, which is sometimes phrased as follows: waves and particles are two distinct types of complementarity in nature, and the experimental situation determines the particle or wave nature of a quantum system; however, the simultaneous observation of wave and particle behavior is impossible. Mutual exclusiveness is regarded by Bohr as a "necessary" element in the complementarity principle to ensure its inner consistency [3]. The usual discussion about wave-particle duality starts from a physical system with two alternatives, typically, a two-way interferometer such as Young's double-slit experiment or a Mach-Zehnder setup. If one performs quantum measurements to determine which way a quantum particle is taken (particle-like property), the interference pattern (wave-like property) is partially or completely destroyed by the partial or complete knowledge of the "which-way" information. The more one obtains the which-way information, the more the loss of interference [4–8]. Many experiments have demonstrated this complementarity with different quantum systems, like atoms [9], lasers [10], nuclear magnetic resonance [11], and single photons [12–15]. Obviously, the concept of measurement plays an important role in a logically consistent description of the wave-particle properties in a two-way interferometer.

Bohr's interpretation of the interferometry experiments in-

vokes the concept of wave-function collapse. Measurements perturb the wave function, so the collapse hypothesis is responsible for mutually exclusive quantities. Based on classical concepts and intuitions, Bohr thought that the stages of preparation and registering the quantum objects require classical apparatuses, which draw an obvious border between the quantum and the classical world. However, this concept is contrary to the belief that quantum theory is universally applicable and classical reality may be reconstructed or reconstituted from quantum dynamics. By treating the apparatus as a large number of particles or a large number of degrees of freedom obeying the Schrödinger equation, the loss of interference is explained by the nonseparable correlation between the measuring apparatus and the system being observed, where the information of the measured system is stored in the pointer states [16, 17] of the apparatus. Such a correlation between two systems is called entanglement. Mathematically, the loss of interference is described by the elimination of the off-diagonal elements of the system's reduced density matrix, which is called decoherence in the terminology of quantum measurement theory. By utilizing entanglement, the position-momentum uncertainty relation is found to play no role in the principle of complementarity [7, 18]. Note that a quantitative formulation of Bohr's complementarity principle can be derived from the position-momentum uncertainty [4, 19]. To find out what makes the quantum apparatus to have a number of states equal to a number of possible distinct outcomes of the measurement, an environment is necessarily introduced to interact with the quantum apparatus [20] for the interferometry experiments.

In the duality relation [7], the wave nature is described in terms of the visibility of the interference pattern. The particle nature is characterized by the path distinguishability, which is a measure of the which-way information. The wave-particle duality is demonstrated by the visibility of fringes setting limits on the which-way information. With the advent

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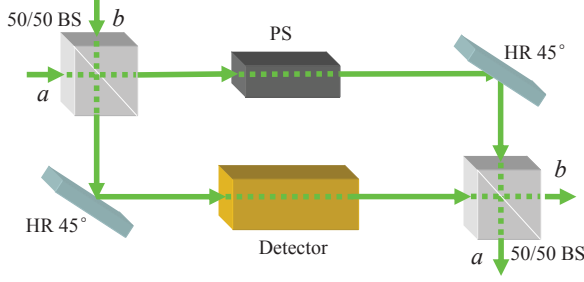


FIG. 1: (Color online). Schematic of a symmetric Mach-Zehnder interferometer. Here, *PS* refers to phase shifter, *BS* to beam splitters, and *HR* to high reflector.

of quantum information theory, there are many measurements which can quantify how much information about a system *A* is stored inside the other system *B*, including both von Neumann [21] and weak measurements [22–24]. Classical correlations (CC) [25, 26] and Quantum discord (QD) [27, 28] are based on von Neumann measurements and are originally introduced as an information-theoretic approach to decoherence mechanisms in a quantum measurement process. The close link between the measurement process and quantitative analysis of the complementarity suggests that CC (QD) and which-way information (visibility) might be different views of one single phenomenon. In this paper, we investigate the relationship between the amount of information that can be extracted about the measurement process and interference in a which-way experiment.

This paper is organized as follows. In Sec. II, we briefly review the well-known method of quantifying wave-particle duality. In Sec. III, we study the amount of information gain from the quantum measurement process by using modern quantum measurement theory in a which-way experiment. The relationship between the amount of information gain and the fringe visibility are also investigated. Finally, We conclude this work in Sec. IV.

II. WAVE-PARTICLE DUALITY RELATION

A symmetric Mach-Zehnder interferometer has two 50:50 beam splitters (BSs) and a phase shifter (PS) as shown in Fig. 1. Between the two BSs, two possible routes *a* and *b* are macroscopically well separated, which are represented by orthogonal unit vectors $|a\rangle$, $|b\rangle$ of a two dimensional Hilbert space. Hereafter, states $|a\rangle$ and $|b\rangle$ are called path states. The wave-function of a photon incident on path *a* (*b*) is changed to an equally weighted superposition

$$|a\rangle \rightarrow \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle) \quad (1a)$$

$$|b\rangle \rightarrow \frac{1}{\sqrt{2}}(|a\rangle - |b\rangle) \quad (1b)$$

by the first BS. As a single photon propagates along this path, a relative phase ϕ is accumulated between states $|a\rangle$ and $|b\rangle$. To obtain the knowledge of the actual path a photon has taken,

a which-way detector (WWD) is introduced, which performs a quantum non-demolition measurement without any backaction on this photon. If a photon propagates along a path *a*, the initial state ρ_{in}^D of the WWD remains unchanged; however, ρ_{in}^D is changed to $U\rho_{\text{in}}^D U^\dagger$ if the photon propagates on path *b*. States ρ_{in}^D and $U\rho_{\text{in}}^D U^\dagger$ are not always orthogonal. By defining the operators σ_α ($\alpha = x, y, z$) in terms of the states $|a\rangle$ and $|b\rangle$, e.g., $\sigma_z = |a\rangle\langle a| - |b\rangle\langle b|$, the degree of freedom described by the path states is analogous to a spin. Therefore, the initial state of a photon is generally characterized by the density matrix

$$\rho_{\text{in}}^Q = \frac{1}{2}(1 + S_x\sigma_x + S_y\sigma_y + S_z\sigma_z) \quad (2)$$

with an initial Bloch vector $\vec{S} = (S_x, S_y, S_z)$. The initial product state $\rho_{\text{in}}^Q \otimes \rho_{\text{in}}^D$ of the two subsystems is evolved into

$$\begin{aligned} \rho_f = & \frac{1}{4}(1 - S_x)(1 + \sigma_x) \otimes \rho_{\text{in}}^D \\ & + \frac{1}{4}(1 + S_x)(1 - \sigma_x) \otimes U\rho_{\text{in}}^D U^\dagger \\ & - \frac{1}{4}e^{-i\phi}(S_z - iS_y)(\sigma_z - i\sigma_y) \otimes \rho_{\text{in}}^D U^\dagger \\ & - \frac{1}{4}e^{i\phi}(S_z + iS_y)(\sigma_z + i\sigma_y) \otimes U\rho_{\text{in}}^D, \end{aligned} \quad (3)$$

after the photon has gone through the Mach-Zehnder interferometer, which establishes the correlation between the photon and the WWD. Then, the which-way information is stored in the WWD. The state of the single photon reads

$$\begin{aligned} \rho_f^Q = & \frac{1}{4}(1 - S_x)(1 + \sigma_x) + \frac{1}{4}(1 + S_x)(1 - \sigma_x) \\ & - \frac{1}{4}e^{-i\phi}(S_z - iS_y)(\sigma_z - i\sigma_y) \text{Tr}_D(\rho_{\text{in}}^D U^\dagger) \\ & - \frac{1}{4}e^{i\phi}(S_z + iS_y)(\sigma_z + i\sigma_y) \text{Tr}_D(U\rho_{\text{in}}^D) \end{aligned} \quad (4)$$

by tracing over the degrees of freedom of the WWD. The probability for the photon emerging from the output *a*

$$\begin{aligned} P^a = & \text{Tr}_Q \left[\frac{1}{2}(1 + \sigma_z)\rho_f^Q \right] \\ = & \frac{1}{2} - \frac{1}{2}\sqrt{S_y^2 + S_z^2} |\text{Tr}_D(U\rho_{\text{in}}^D)| \cos(\alpha + \beta + \phi) \end{aligned} \quad (5)$$

is used to define the visibility

$$V \equiv \frac{P_{\text{max}}^a - P_{\text{min}}^a}{P_{\text{max}}^a + P_{\text{min}}^a} = \sqrt{S_y^2 + S_z^2} |\text{Tr}_D(U\rho_{\text{in}}^D)| \quad (6)$$

of the interference pattern, where the constant phase shifts α and β are the phases of $S_z + iS_y$ and $\text{Tr}_D(U\rho_{\text{in}}^D)$ respectively. To extract the information in the final state of the WWD

$$\rho_f^D = \frac{1 - S_x}{2}\rho_{\text{in}}^D + \frac{1 + S_x}{2}U\rho_{\text{in}}^D U^\dagger, \quad (7)$$

an observable must be chosen for the readout. Englert [7] introduced the distinguishability

$$D = \text{Tr}_D \left| \frac{1 - S_x}{2}\rho_{\text{in}}^D - \frac{1 + S_x}{2}U\rho_{\text{in}}^D U^\dagger \right| \quad (8)$$

to be the maximum of the difference of probabilities of the correct and incorrect decisions about the paths. Then the fringe visibility and the maximum amount of which-way information are bound in a trade-off relation

$$D^2 + \frac{1 - P^2}{V_0^2} V^2 \leq 1, \quad (9)$$

where $P = |S_x|$ is the predictability and $V_0 = \sqrt{S_y^2 + S_z^2}$ is a priori fringe visibility. Actually, the parameters P and V_0 construct a trade-off relation $P^2 + V_0^2 \leq 1$, which is known as the duality relationship for preparation [5].

Special attention is paid in Ref. [7] on the initial state with $S_x = 0$ and $S_z + iS_y = e^{-i\theta}$ (in this case, $P = 0$, $V_0 = 1$) to emphasize the quantum properties of the WWD, which enforce duality and make sure that the principle of complementarity is not circumvented. In this sense, we will set $S_x = 0$ and $S_z + iS_y = e^{-i\theta}$ in the rest of our paper.

III. INFORMATION GAIN VERSUS INTERFERENCE

A. Traditional approach

In order to acquire the which-way information, a WWD is introduced to interact with the photon, and a suitable observable must be chosen for the readout. For simplicity, we take the initial state of the WWD as a pure state, i.e., $\rho_{\text{in}}^D = |d\rangle\langle d|$. Reference [7] relies on von Neumann's notion of quantum measurement, i.e., the WWD interacts with the photon and becomes entangled with it, and the state of the combined photon-detector system can then be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle \otimes |d\rangle + |b\rangle \otimes U|d\rangle). \quad (10)$$

Next, the ‘‘likelihood for guessing the way right’’ is introduced [7] to describe the which-way information. And the largest amount of information can be obtained when the eigenstates of the observable are also the eigenstates of the difference $|d\rangle\langle d| - U|d\rangle\langle d|U^\dagger$. From Eqs. (6, 8), the visibility of the interference pattern $V = |\langle d|U|d\rangle|$, and the path distinguishability $D = \sqrt{1 - |\langle d|U|d\rangle|^2}$, which satisfy the complementarity principle $V^2 + D^2 = 1$. We note that the above strategy is equal to the theory of quantum state discrimination with minimum error [29, 30]. The error-minimum discrimination is described by the projection operators $\Pi_A = |M_A\rangle\langle M_A|$ and $\Pi_B = |M_B\rangle\langle M_B|$, where optimum measurement vectors are given by

$$\begin{aligned} |M_A\rangle &= \frac{1}{m} \sqrt{\frac{1+m}{2}} |d\rangle - e^{i\varphi} \frac{1}{m} \sqrt{\frac{1-m}{2}} U|d\rangle, \\ |M_B\rangle &= e^{-i\varphi} \frac{1}{m} \sqrt{\frac{1-m}{2}} |d\rangle - \frac{1}{m} \sqrt{\frac{1+m}{2}} U|d\rangle \end{aligned} \quad (11)$$

with $m = \sqrt{1 - V^2}$, and φ being the relative phase. In fact, the optimum measurement vectors $|M_A\rangle$ and $|M_B\rangle$ are exactly the eigenstates of the difference $|d\rangle\langle d| - U|d\rangle\langle d|U^\dagger$.

B. Classical correlations versus interference

Here we try to evaluate the which-way information on a different point of view with the aid of quantum information theory. In quantum information theory, the classical correlations between the photon and the WWD can be captured by

$$\mathcal{J}(\rho_f') = \max [S(\rho^Q) - S(\rho^Q|\{\Pi_k\})], \quad (12)$$

where ρ^Q is the reduced density operator for the photon, $S(\rho^Q)$ is the von Neumann entropy, $S(\rho^Q|\{\Pi_k\})$ is the quantum conditional entropy, and $\{\Pi_k\}$ is a set of projectors performed locally on the WWD. To calculate the quantum conditional entropy, we choose the von Neumann projection, and the orthogonal projection operators $\Pi_1 = |M_1\rangle\langle M_1|$ and $\Pi_2 = |M_2\rangle\langle M_2|$, where $|M_1\rangle$ and $|M_2\rangle$ are given by

$$\begin{aligned} |M_1\rangle &= \frac{\sin \gamma}{\sqrt{1 - V^2}} |d\rangle + e^{i\varphi} \left(\cos \gamma - \sin \gamma \frac{V}{\sqrt{1 - V^2}} \right) U|d\rangle, \\ |M_2\rangle &= e^{-i\varphi} \frac{\cos \gamma}{\sqrt{1 - V^2}} |d\rangle - \left(\sin \gamma + \cos \gamma \frac{V}{\sqrt{1 - V^2}} \right) U|d\rangle \end{aligned} \quad (13)$$

with $V = |\langle d|U|d\rangle|$, $0 \leq \varphi \leq 2\pi$ and $0 \leq \gamma \leq 2\pi$. Actually, our strategy to discriminate two non-orthogonal states of the WWD and obtain the which-way information is considering a set of von Neumann projection performed locally on the WWD. The CC is the largest classical mutual information gained about photon after a measurement of the WWD.

(1) *Based on von Neumann's notion of quantum measurement*

The state of the combined photon-detector system in Eq. (10) can be written as

$$\rho' = |\Psi\rangle\langle\Psi| = \frac{1}{2} \begin{pmatrix} 1 & 0 & V & \sqrt{1 - V^2} \\ 0 & 0 & 0 & 0 \\ V & 0 & V^2 & V\sqrt{1 - V^2} \\ \sqrt{1 - V^2} & 0 & V\sqrt{1 - V^2} & 1 - V^2 \end{pmatrix} \quad (14)$$

in the basis $\{|a\rangle|d\rangle, |a\rangle|d_\perp\rangle, |b\rangle|d\rangle, |b\rangle|d_\perp\rangle\}$. After a straightforward calculation, we can obtain the CC

$$\mathcal{J}(\rho') = -\frac{1+V}{2} \log\left(\frac{1+V}{2}\right) - \frac{1-V}{2} \log\left(\frac{1-V}{2}\right), \quad (15)$$

which is shown by the blue-solid curve in Fig. 2.

(2) *Based on Zurek's ‘‘triple model’’ of quantum measurement*

To realize the wave-function collapse of the measured system by establishing an entanglement between the system and the apparatus in quantum measurement theory, an observer must first select the state of detector and then read it out. To avoid this subjective selection, Zurek introduced a ‘‘triple model’’ of quantum measurement process, which consists of a measured system (photon), an apparatus (WWD), and an environment. According to the environment-induced superselection, the entangled state of the combined photon-detector system becomes a correlated state

$$\rho'' = \frac{1}{2} (|a\rangle\langle a| \otimes |d\rangle\langle d| + |b\rangle\langle b| \otimes U|d\rangle\langle d|U^\dagger), \quad (16)$$

which can be written as

$$\rho'' = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & V^2 & V\sqrt{1-V^2} \\ 0 & 0 & V\sqrt{1-V^2} & 1-V^2 \end{pmatrix} \quad (17)$$

in the basis $\{|a\rangle|d\rangle, |a\rangle|d_\perp\rangle, |b\rangle|d\rangle, |b\rangle|d_\perp\rangle\}$. It can be found that the state ρ'' is no longer an entangled state, and the state $|d\rangle$ ($U|d\rangle$) becomes correlated with $|a\rangle$ ($|b\rangle$). We note that the states $|d\rangle$ and $U|d\rangle$ are two pointer states of the WWD. If the state of the WWD is $|d\rangle$ ($U|d\rangle$), the observer can infer that the photon passes through the path a (b). Note that the observer just reads out the pointer states of WWD. If $|d\rangle = U|d\rangle$,

the two pointer states of the WWD are the same. Two equal pointer states cannot indicate two possible outcomes. Therefore, the paths of the photon cannot be distinguished, and we cannot obtain the which-way information, i.e., $D = 0$. If $|d\rangle$ and $U|d\rangle$ are mutually orthogonal, the two pointer states are completely different and can be perfectly discriminated. Two different pointer states correspond to two different paths of the photon, and a perfect distinction of the photon path is achieved when $D = 1$. However, when $|d\rangle$ and $U|d\rangle$ are not mutually orthogonal, it is impossible to discriminate them perfectly.

According to the Eq. (17), after a straightforward calculation, we obtain

$$\begin{aligned} S(\rho^\mathcal{Q}) - S(\rho^\mathcal{Q}|\{\Pi_k\}) &= 1 + \frac{1}{2} [\cos^2 \gamma \log [\cos^2 \gamma] + \frac{1}{2} [\sin^2 \gamma \log [\sin^2 \gamma] \\ &\quad + \frac{1}{2} [(\sqrt{1-V^2} \cos \gamma - V \sin \gamma)^2 \log [(\sqrt{1-V^2} \cos \gamma - V \sin \gamma)^2] \\ &\quad + \frac{1}{2} [(\sqrt{1-V^2} \sin \gamma + V \cos \gamma)^2 \log [(\sqrt{1-V^2} \sin \gamma + V \cos \gamma)^2] \\ &\quad - \frac{1}{2} [(\sqrt{1-V^2} \sin \gamma + V \cos \gamma)^2 + \cos^2 \gamma \log [(\sqrt{1-V^2} \sin \gamma + V \cos \gamma)^2 + \cos^2 \gamma] \\ &\quad - \frac{1}{2} [(\sqrt{1-V^2} \cos \gamma - V \sin \gamma)^2 + \sin^2 \gamma \log [(\sqrt{1-V^2} \cos \gamma - V \sin \gamma)^2 + \sin^2 \gamma]. \end{aligned} \quad (18)$$

We find that $S(\rho^\mathcal{Q}) - S(\rho^\mathcal{Q}|\{\Pi_k\})$ is a periodic function of the angle γ , and its cycle is $\pi/2$. So, in order to obtain its maximum value, we just select the appropriate value of γ in the range of $0 \leq \gamma \leq \pi/2$ for a given V . In principle, it is difficult to derive the analytical expression of the maximum value because it involves a transcendental equation. Here, we adopt a more rudimentary and primitive method. As $S(\rho^\mathcal{Q}) - S(\rho^\mathcal{Q}|\{\Pi_k\})$ is a function of both the angle γ and the visibility V , we can fix one parameter V and then find what value of γ maximizes $S(\rho^\mathcal{Q}) - S(\rho^\mathcal{Q}|\{\Pi_k\})$. So, the corresponding relationship between the parameters γ and V can be revealed. It can be found that the angle γ is a inverse trigonometric function about V , i.e.,

$$\gamma = \arcsin \left[\sqrt{\frac{1 + \sqrt{1-V^2}}{2}} \right]. \quad (19)$$

Substituting this value of γ into Eq. (18), we can obtain the CC

$$\begin{aligned} \mathcal{J}(\rho'') &= \frac{1 + \sqrt{1-V^2}}{2} \log(1 + \sqrt{1-V^2}) \\ &\quad + \frac{1 - \sqrt{1-V^2}}{2} \log(1 - \sqrt{1-V^2}), \end{aligned} \quad (20)$$

which is shown by the red-dashed curve in Fig. 2.

From the Fig. 2, it can be observed that when $V = 1$, $CC = 0$, and when $V = 0$, $CC = 1$. For the former

case, since the two pointer states of the WWD are the same, i.e., $|d\rangle = U|d\rangle$, we cannot distinguish the two paths of the photon by discriminating the two pointer states. Moreover, both classical correlations and distinguishability are equal to zero, i.e., $CC = D = 0$, while the fringe visibility $V = 1$, which means that the wave-like behavior of the photon can be perfectly observed. For the latter case, when two pointer states $|d\rangle$ and $U|d\rangle$ are mutually orthogonal, the two paths of the photon can be distinguished, and both classical correlations and distinguishability reach the maximum values, i.e., $CC = D = 1$. However, the wave-like behavior of the photon will disappear, i.e., $V = 0$. When $0 < V < 1$, the photon is in a superposition. As the fringe visibility V increases, the CC decays monotonously, while the distinguishability satisfies $D = \sqrt{1-V^2}$, which decays monotonously with the visibility V as well. Actually, since the two pointer states of the WWD overlap, we can just partly distinguish the two paths of the photon, while an imperfect interference fringe is exhibited. According to the above analysis, like the distinguishability, to some extent, the CC can be also used to describe the which-way information of the photon. The more we obtain the which-way information (the CC), the smaller the fringe visibility V will be. Furthermore, Eq. (15, 20) provide the complementary relation between the fringe visibility and the CC. Indeed, the definitions of the distinguishability and CC have a common basic-orthogonal projection. We note that the definition of distinguishability is related to the orthogonal projec-

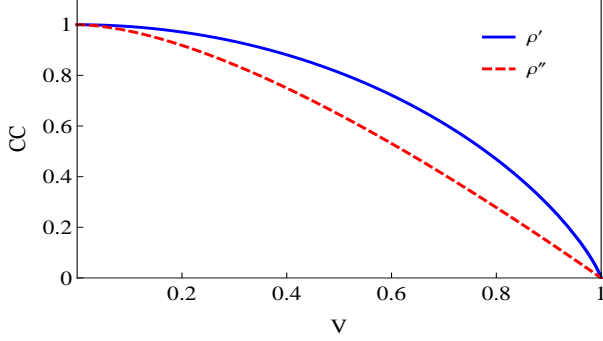


FIG. 2: (Color online). The relationship between the classical correlations and the fringe visibility V .

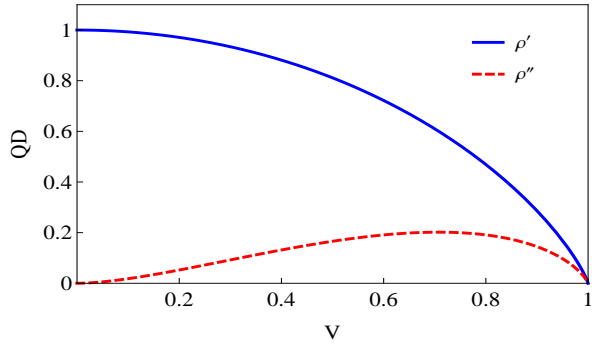


FIG. 3: (Color online). The relationship between the quantum correlations and the fringe visibility V .

tion vectors $|M_A\rangle$ and $|M_B\rangle$ which are expressed as in Eq. (11), while the definition of the CC is related to the orthogonal projection vectors $|M_1\rangle$ and $|M_2\rangle$ in Eq. (14) with the condition $\gamma = \arcsin\left[\sqrt{\frac{1+\sqrt{1-V^2}}{2}}\right]$. After a simple calculation and contrast, we can find that the two sets of orthogonal projections are the same. Then, it is natural to use the CC to describe the which-way information of the photon. In addition, because of the consistency between the CC and the distinguishability, it is reasonable to conclude that the CC is just the information gained about the particle-like property of the photon, rather than the information gained about the wave-like property of the photon.

C. Quantum correlations versus interference

Quantum discord (QD) was originally introduced as an information-theoretic approach to describe decoherence mechanisms in a quantum measurement process. The wave-like property (the visibility of the interference pattern) of the photon is also related to quantum coherence. Thus, there is a relation between these, which will be considered below. Quantum discord, \mathcal{D} , can be used to measure quantum correlations in a bipartite system, and it is defined as the difference between the total correlations, \mathcal{I} , and the classical

correlations, \mathcal{J} ,

$$\mathcal{D}(\rho) = \mathcal{I}(\rho) - \mathcal{J}(\rho). \quad (21)$$

Here, the total correlations is equal to quantum mutual information

$$\mathcal{I}(\rho) = S(\rho^Q) + S(\rho^D) - S(\rho), \quad (22)$$

where S is the von Neumann entropy, and ρ^Q (ρ^D) is the reduced density matrix of the photon (WWD).

(1) *Based on von Neumann's notion of quantum measurement*

Considering the combined photon-detector system is in the entangled state in Eq. (10), we can obtain the quantum correlations between the photon and the WWD

$$\mathcal{D}(\rho') = -\frac{1+V}{2} \log\left(\frac{1+V}{2}\right) - \frac{1-V}{2} \log\left(\frac{1-V}{2}\right), \quad (23)$$

which is shown by the blue-solid curve in Fig. 3. We can find that the quantum correlations are equal to the classical correlations. In fact, for any pure state, the total correlations are equally divided into the classical and quantum correlations [31]. In this case, the quantum correlations have the same behaviors with the classical correlations.

(2) *Based on Zurek's triple model of quantum measurement*

Considering the combined photon-detector system is in the state ρ'' in Eq. (16), the QD between the photon and the WWD can be written as

$$\begin{aligned} \mathcal{D}(\rho'') = & -\frac{1+V}{2} \log\left(\frac{1+V}{2}\right) - \frac{1-V}{2} \log\left(\frac{1-V}{2}\right) \\ & - \frac{1+\sqrt{1-V^2}}{2} \log\left(1+\sqrt{1-V^2}\right) \\ & - \frac{1-\sqrt{1-V^2}}{2} \log\left(1-\sqrt{1-V^2}\right). \end{aligned} \quad (24)$$

We plot the QD as a function of the fringe visibility V as shown in the red-dashed curve in Fig. 3. It can be seen that the QD is not a monotonic function of the fringe visibility V . When $V = 0$, we have $QD = 0$, and $CC = 1$. Since the two pointer states of the WWD are mutually orthogonal, a perfect discrimination among the two paths of the photon can be achieved. From the point of view of quantum information theory, there are only classical correlations and no quantum correlations between photon and WWD, and such state of the combined photon-detector system is a classical state [32, 33]. Here, we note that when one makes a perfect quantum measurement, the state of the combined photon-detector system should be a classical state. In this case, we can confirm that no quantum correlations correspond to no fringe visibility. As V increases, the QD first increases and then decreases. In this process, since the two pointer states of the WWD have an overlap, the state of the combined photon-detector system becomes a separable state [34]. Although there is no entanglement in a separable state, some classical correlations (CC) and quantum correlations (QD) may exist. Here, we can see that CC is used to describe the which-way information, but QD is

different from the fringe visibility. When $V = 1$, both CC and QD are zero. Since the two pointer states of the WWD are the same, from Eq. (16) the state of the combined photon-detector system is changed to a product state or an uncorrelated state. In this case, the QD is equal to zero; however, the fringe visibility $V = 1$. Obviously, the QD is different from the fringe visibility V , though both of these are related to quantum coherence.

IV. DISCUSSION AND CONCLUSION

In the quantitative relation formulation of wave-particle duality, the wave nature is described by the visibility of the interference pattern, while the particle nature is characterized by the path distinguishability, which is based on von Neumann's measurement theory. In modern measurement theory, to acquire the which-way information about a particle traveling, a "triple model" (which consists of a measured system, an apparatus and environment) must be introduced. Here, we use Zurek's "triple model" to study wave-particle duality in a symmetric Mach-Zehnder interferometer, where the interaction between the quantum system and the apparatus produces a quantum entanglement between them, and later the coupling of the environment and the apparatus generates a triple entanglement among the system, apparatus and environment. By tracing the environment, the state of system and apparatus

is no longer an entangled state, but a correlated state. With the help of quantum information theory, it is easy to see that the correlations between the measured system and detector [which include classical correlations (CC) and quantum correlations (QD)] are related to the information gain about the measured system. It is found that the CC is the information gain about the particle-like property of the measured system which is consistent with the path distinguishability. Moreover, we can also see that QD and the visibility have different values, but both of them represent one single phenomenon—quantum coherence. Finally, we derive an analytical expression for the QD of one type of two-qubit separable states, i.e., quantum-classical states in Eq. (17). Since the analytical expression for the QD can be written only for some special type of two-qubit states at present [31], our analysis broadens the regime of analytical expressions for the QD of two-qubit states.

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